Non-adaptive Group Testing on Graphs

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Abstract

Grebinski and Kucherov (1998) and Alon et al. (2004-2005) study the problem of learning a hidden graph for some especial cases, such as hamiltonian cycle, cliques, stars, and matchings. This problem is motivated by problems in chemical reactions, molecular biology and genome sequencing.

In this paper, we present a generalization of this problem. Precisely, we consider a graph G and a subgraph H of G and we assume that G contains exactly one defective subgraph isomorphic to H. The goal is to find the defective subgraph by testing whether an induced subgraph contains an edge of the defective subgraph, with the minimum number of tests. We present an upper bound for the number of tests to find the defective subgraph by using the symmetric and high probability variation of Lovász Local Lemma.

Keywords: Group testing on graphs, Non-adaptive algorithm, Combinatorial search, Learning hidden subgraph.

1 Introduction

In the classic *group testing* problem that was first introduced by Dorfman[9], there is a set of n items which contain at most d defective items. The goal of this problem is to find the defective items with the minimum number of tests. Every test consists of some items and each test is positive if it contains at least one defective item and otherwise, test is negative. There are two types of algorithms for group testing problem, *sequential* and *non-adaptive*. In the sequential algorithm, the outcome of previous tests can be used in the future tests and in non-adaptive algorithm all tests perform in the same time and by the result of all tests we should find the defective items. Group testing has many applications including finding pattern in data, DNA library screening, and so on, for an overview of results and more applications, we refer the reader to [10, 11, 15].

Aigner [1] proposed the problem of *group testing on graphs*. In this problem, we are looking for one defective edge of the given graph G by performing the minimum sequential tests, where each test is an induced subgraph of the graph G and the test is positive if it contains the defective edge.

In this paper, we consider the problem of *non-adaptive group testing on graphs*. Suppose that there is a defective subgraph (not necessarily induced subgraph) of G isomorphic to a graph H. Each test F is an induced subgraph of G and the outcome of test is positive if and only if F has at least one edge in common with the defective subgraph. This is a generalization of the problem of learning hidden subgraph posed in [12, 2, 3]. More precisely, in learning hidden subgraph problem, the graph G is a complete graph, see [12] for some alternative formulation.

This problem has many applications in chemical reactions, molecular biology and genome sequencing. In chemical reactions, we have a set of chemicals, some pairs of them may be have a reaction. Moreover, before testing we know some pairs have no reaction. When some chemicals combined in one test, a reaction occurred if and only if at least on pair of the chemicals in the test reacts. Our goal is to identify which pairs react using as few tests as possible. We can reformulate this problem as follows. Suppose that there are n vertices and two vertices u and v are adjacent if and only if two chemicals u and v may be have a reaction. Each pair of chemicals where have a reaction shows a defective edge and finding all these type of pairs is equal to find the defective subgraph. Since we are aware that some pairs have no reaction, the graph G is not necessarily a complete graph.

There are various families of hidden graphs to study. Many recent studies focus on two cases: Hamiltonian cycles and matchings [3, 6, 12] which have specific application to the genome sequencing and DNA physical mapping. For more information about these applications and their results refer to [5, 7, 12].

2 Notation

Throughout this paper, we suppose that H is a subgraph of G with k edges. Moreover, we assume that G contains exactly one defective subgraph isomorphic to H. We denote the maximum degree of H by $\Delta = \Delta(H)$. Also, $G[X]$ denotes the subgraph of G induced by $X \cap V(G)$ and for any vertex $v \in G$, $N_H(v)$ stands for the set of neighbours of the vertex v in the graph H. Hereafter, we assume that the subgraph H has no isolated vertex, because in the problem of group testing on graphs, vertices are not defective.

A boolean matrix is said to be *d-disjunct* if for every column C_0 and every choice of d columns $C_1, C_2, \ldots, C_d$ (different from C_0), there is at least one row such that the entry corresponding to C_0 is 1 and the entries corresponding to $C_1, C_2, \ldots, C_d$ are all zeros. This concept was first introduced in [14].

3 Main result

For $1 \leq l \leq t$, let $\mathcal{F}_l$ be a random test obtained by choosing each vertex of $V(G)$ independently with probability p. For simplicity of notation we write F_i as an induced subgraph of G on vertices of $\mathcal{F}_i$.

Throughout this paper, let $H_1, H_2, \ldots, H_m$ be all the subgraphs of G isomorphic to H. Let C be a random $t \times m$ matrix such that for any l and j, where $1 \leq j \leq m$ and $1 \leq l \leq t$, if $E(F_l \cap H_j) \neq \varnothing$, then $c_{lj} = 1$; otherwise, $c_{lj} = 0$. The lth row of this matrix corresponds to the test F_l and the jth column corresponds to the subgraph H_j. For any i, j, l, where $1 \leq i \neq j \leq m$ and $1 \leq l \leq t$, define the event $A^l_{i,j}$ to be the set of all matrices C such that $c_{li} \leq c_{lj}$. Also, define the event $A_{i,j}$ to be the set of all matrices C such that for every l, $1 \leq l \leq t$, we have $c_{li} \leq c_{lj}$. In other words, if the event $A^l_{i,j}$ occurs, then the test F_l cannot distinguish between H_i and H_j. Also, if the event $A_{i,j}$ occurs, then for every l such that $1 \leq l \leq t$, the test F_l cannot distinguish between H_i and H_j. We would like to bound the

probability that none of the bad events $A_{i,j}$ occur. In such cases, when there is some relatively small amount of dependence between events, one can use a powerful generalization of the union bound, known as the Lovász Local Lemma. The main device in establishing the Lovász Local Lemma is a graph called the dependency graph. Let $A_1, A_2, \ldots, A_n$ be events in an arbitrary probability space. A graph $D = (V, E)$ on the set of vertices $V = \{1, 2, \ldots, n\}$ is a dependency graph for events $A_1, A_2, \ldots, A_n$ if for each $1 \leq i \leq n$ the event A_i is mutually independent of all the events $\{A_j : \{i, j\} \notin E\}$. we state the Lovász Local Lemma as follows.

Lemma A. [4] (*Lovász Local Lemma, Symmetric Case*). *Suppose that $A_1, A_2, \ldots, A_n$ are events in a probability space with $Pr(A_i) \leq p$ for all i. If the maximum degree in the dependency graph of these events is d, and if $ep(d+1) \leq 1$, then*

$$Pr\Big(\bigcap_{i=1}^{n} \overline{A_i}\Big) > 0,$$

where e is the basis of the natural logarithm.

To find the maximum degree in the dependency graph of the events $A_{i,j}$, we define the parameter $r_G(H)$ as follows. Set $r_G(H, H_i)$ is the number of subgraphs isomorphic to H whose intersection with H_i is nonempty and define $r_G(H) = \max_i r_G(H, H_i)$.

In the main theorem, we show that the aforementioned random matrix is a 1-disjunct matrix with positive probability. A well known theorem stated in [13], asserts that to find the defective items, it is sufficient to create a disjunct matrix. More precisely, in the main theorem, we prove there is a $t \times m$ matrix C such that for every i and j, there are two distinct numbers $1 \leq l, l' \leq t$, such that $C_{l,i} = 1,\ C_{l,j} = 0$ and $C_{l',i} = 0,\ C_{l',j} = 1$. So if H_i is a defective subgraph, then for every non-defective subgraph H_j, there exists a test F_l such that $E(F_l) \cap E(H_i) = \varnothing$ and $E(F_l) \cap E(H_j) \neq \varnothing$. So the test F_l distinguish between the defective subgraph H_i and every non-defective subgraph H_j. Therefore, by this matrix we can find every non-defective subgraph isomorphic to H.

Theorem 1. *Let H be the defective subgraph with $E(H) = k$, $\Delta(H) = \Delta$. One can find the subgraph H with t non-adaptive tests, where*

$$t = 1 + \left\lceil \frac{\ln(4er_G(H)) + \ln m}{\ln \frac{1}{1-P_{k,\Delta}}} \right\rceil,$$

$P_{k,\Delta} = \frac{1}{2k\Delta}\left(1 - \frac{1}{2\Delta}\right)^{2\Delta-1}\left(1 - \sqrt{\frac{1}{2k\Delta}}\left(1 - \frac{1}{2\Delta}\right)^{\Delta-1}\right)^{2\Delta-2}$, *and e is the basis of the natural logarithm.*

To prove the main theorem, we need some supportive results.

Lemma 1. *Let H be a graph with n vertices, k edges, and maximum degree Δ. Pick, randomly and independently, each vertex of H with probability p, where $p = \sqrt{\frac{\epsilon}{k}}(1 - \frac{\epsilon}{k})^{(\Delta-1)}$. If F is the set of all chosen vertices, then $H[F]$ is an independent set with probability at least $1 - \epsilon$.*

To prove this lemma, we need high probability variation of Lovász Local Lemma.

Lemma B. [8] *Let $B_1, B_2, \dots, B_k$ be events in a probability space. Suppose that each event B_i is independent of all the events B_j but at most d. For $1 \le i \le k$ and $0 < \epsilon < 1$, if $Pr(B_i) \le \frac{\epsilon}{k}(1-\frac{\epsilon}{k})^d$, then $Pr\Big(\bigcap_{i=1}^{k} \overline{B_i}\Big) > 1-\epsilon$.*

Proof of Lemma 1. Let $E(H) = \{e_1, e_2, \dots, e_k\}$. For $1 \le i \le k$, we define B_i to be the event that $e_i \in E(H[F])$, so $Pr(B_i) = p^2$. Since vertices are chosen randomly and independently, the event B_i is independent of the event B_j if and only if edges e_i and e_j have no common vertex. So the maximum degree of the dependency graph is at most $2(\Delta-1)$. Since $p^2 \le \frac{\epsilon}{k}\left(1-\frac{\epsilon}{k}\right)^{2(\Delta-1)}$, by Lemma B, $Pr\Big(\bigcap_{i=1}^{k} \overline{B_i}\Big) > 1-\epsilon$. Hence, $H[F]$ is an independent set with probability at least $1-\epsilon$. ■

The problem of finding the probability that tests $F_1, F_2, \dots, F_t$, distinguish between every pair of subgraphs H_i and H_j, falls into different lemmas as follows.

Lemma 2. *If $V(H_i) = V(H_j)$ and $|E(H_i) \setminus E(H_j)| = 1$, then*

$$Pr\big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\big) \ge p^2(1-p)^{2\Delta}(1-\epsilon),$$

where $p = \sqrt{\frac{\epsilon}{k}}\,(1-\epsilon)^{\Delta-1}$.

Proof. Let $e = \{u, v\} \in E(H_i) \setminus E(H_j)$. Consider the induced subgraph H', where $V(H') = V(H_j) \setminus \Big(u \cup v \cup N(u) \cup N(v)\Big)$. Note that $E(F_l \cap H_i) \neq \varnothing$ and $E(F_l \cap H_j) = \varnothing$ if and only if $H_j \cap F_l$ is an independent set of H_j and $u, v \in \mathcal{F}_l$. Also, one can see that $u, v \in \mathcal{F}_l$ and $H_j[F_l]$ is an independent set if and only if the following events hold

1. $u, v \in \mathcal{F}_l$,
2. $N_{H_j}(u) \cap \mathcal{F}_l = \varnothing$ and $N_{H_j}(v) \cap \mathcal{F}_l = \varnothing$,
3. $H'[F_l]$ is an independent set.

It is straightforward to check that the aforementioned events are independent. Also, one can see that the event $u, v \in \mathcal{F}_l$ occurs with probability p^2. Since $|N_{H_j}(u) \cup N_{H_j}(v)| \le 2\Delta$,

$$\begin{aligned} Pr\left(N_{H_j}(u) \cap \mathcal{F}_l = \varnothing \;\&\; N_{H_j}(v) \cap \mathcal{F}_l = \varnothing\right) &= \\ Pr\Big(\mathcal{F}_l \cap \big(N_{H_j}(u) \cup N_{H_j}(v) \setminus \{u, v\}\big) = \varnothing\Big) &\ge (1-p)^{2\Delta}. \end{aligned}$$

Set $E(H') = k'$. If $k' = 0$, then $F_l \cap H'$ has no edges. So $Pr\big(E(F_l \cap H') = \varnothing\big) = 1$. Suppose that $k' \ge 1$. Since $k \ge k'$, we have $p^2 = \frac{\epsilon}{k}(1-\epsilon)^{2\Delta-2} \le \frac{\epsilon}{k'}(1-\frac{\epsilon}{k'})^{2\Delta-2}$. Each vertex of the induced subgraph H' is chosen with probability p. So by Lemma 1, the induced subgraph on $\mathcal{F}_l \cap V(H')$ is an independent set with probability at least $1-\epsilon$.

In other words, $Pr\big(E(F_l \cap H') = \varnothing\big) \geq 1 - \epsilon$. Since the events are independent, we have

$$Pr\big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\big) \geq p^2(1-p)^{2\Delta}(1-\epsilon),$$

as desired. ■

Lemma 3. *If $|V(H_i) \setminus V(H_j)| = 1$, then*

$$Pr\Big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\Big) \geq p^2(1-p)^{\Delta}(1-\epsilon),$$

where $p = \sqrt{\frac{\epsilon}{k}}(1-\epsilon)^{\Delta-1}$.

Proof. Since H has no isolated vertex, there exists at least one edge $e = \{u, v\} \in E(H_i) \setminus E(H_j)$. Let $v \in V(H_i) \cap V(H_j)$ and $u \in V(H_i) \setminus V(H_j)$. Suppose that H' is an induced subgraph of H_j, where $V(H') = V(H_j) \setminus (v \cup N(v))$. Set $|E(H')| = k'$. Similar to the proof of Lemma 2, $E(F_l \cap H_i) \neq \varnothing$ and $E(F_l \cap H_j) = \varnothing$ if and only if the following independent events hold

1. $u, v \in \mathcal{F}_l$,
2. $N_{H_j}(v) \cap \mathcal{F}_l = \varnothing$,
3. $H'[F_l]$ is an independent set.

Since $|N_{H_j}(v)| \leq \Delta$, the probability that $N_{H_j}(v) \cap \mathcal{F}_l = \varnothing$ is at least $(1-p)^{\Delta}$. The rest of proof is similar to Lemma 2, so

$$Pr\Big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\Big) \geq p^2(1-p)^{\Delta}(1-\epsilon),$$

as desired. ■

Lemma 4. *If the induced subgraph on $V(H_i) - V(H_j)$ has at least one edge, then*

$$Pr\Big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\Big) \geq p^2(1-\epsilon),$$

where $p = \sqrt{\frac{\epsilon}{k}}(1-\epsilon)^{\Delta-1}$.

Proof. Let $e = (u, v) \in E(H_i) \setminus E(H_j)$. If the following independent events hold

1. $u, v \in \mathcal{F}_l$,
2. $H_j[F_l]$ is an independent set,

then $E(F_l \cap H_i) \neq \varnothing$ and $E(F_l \cap H_j) = \varnothing$. Since $p^2 = \frac{\epsilon}{k}(1-\epsilon)^{2\Delta-2} \leq \frac{\epsilon}{k}\left(1 - \frac{\epsilon}{k}\right)^{2\Delta-2}$, by Lemma 1, $Pr\left(E(F_l \cap H_j) = \varnothing\right) \geq 1 - \epsilon$. Also one can see that

$$Pr\big(E(F_l \cap H_i) \neq \varnothing\big) \geq Pr\big(e \in E(F_l)\big) = Pr(u, v \in \mathcal{F}_l) = p^2.$$

Consequently, $Pr\big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\big) \geq p^2(1-\epsilon)$. ■

In the next theorem and in view of the previous lemmas, we show that the probability of distinguishing between H_i and H_j has the minimum value whenever $V(H_i) = V(H_j)$ and $|E(H_i) \setminus E(H_j)| = 1$.

Theorem 2. *Let $|E(H)| = k$ and $\Delta(H) = \Delta$. For every $1 \leq i \neq j \leq m$ and $1 \leq l \leq t$, we have*

$$Pr\left(\overline{A^l_{i,j}}\right) \geq p^2(1-p)^{2\Delta}(1-\epsilon), \tag{1}$$

where $p = \sqrt{\frac{\epsilon}{k}}\,(1-\epsilon)^{\Delta-1}$.

Proof. Let $E(H_i) \cap E(H_j) = \{f_1, f_2, \ldots, f_r\}$ and $E(H_i) \setminus E(H_j) = \{e_1, e_2, \ldots, e_{k-r}\}$. The event $\overline{A^l_{i,j}}$ occurs if and only if $E(F_l \cap H_i) \neq \varnothing$ and $E(F_l \cap H_j) = \varnothing$. It is easy to check, for every $1 \leq q \leq k - r$,

$$Pr\Big(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\Big) \geq Pr\Big(e_q \in E(F_l \cap H_i), E(F_l \cap H_j) = \varnothing\Big).$$

So we need to consider the following three cases,

case 1: $V(H_i) = V(H_j)$, $|E(H_i) \setminus E(H_j)| = 1$.

By Lemma 2, it is clear $Pr\left(\overline{A^l_{i,j}}\right) \geq p^2(1-p)^{2\Delta}(1-\epsilon)$.

case 2: $|V(H_i) \setminus V(H_j)| = 1$.

By Lemma 3, we have $Pr(\overline{A^l_{i,j}}) \geq p^2(1-p)^{\Delta}(1-\epsilon) \geq p^2(1-p)^{2\Delta}(1-\epsilon)$.

case 3: The induced subgraph on $V(H_i) - V(H_j)$ has at least one edge.

By Lemma 4, $Pr\left(\overline{A^l_{i,j}}\right) \geq p^2(1-\epsilon) \geq p^2(1-p)^{2\Delta}(1-\epsilon)$.

So for every $1 \leq i \neq j \leq m$ and $1 \leq l \leq t$, $Pr\left(\overline{A^l_{i,j}}\right) \geq p^2(1-p)^{2\Delta}(1-\epsilon)$. ■

To prove the main theorem, we present an upper bound for the probability of occurring the bad events $A_{i,j}$ for every $1 \leq i \neq j \leq m$.

Theorem 3. *Let $|E(H)| = k$ and $\Delta(H) = \Delta$. For every $1 \leq i \neq j \leq m$, we have*

$$Pr(A_{i,j}) \leq (1 - P_{k,\Delta})^t, \tag{2}$$

where $P_{k,\Delta} = \frac{1}{2k\Delta}\left(1 - \frac{1}{2\Delta}\right)^{2\Delta-1}\left(1 - \sqrt{\frac{1}{2k\Delta}}\left(1 - \frac{1}{2\Delta}\right)^{\Delta-1}\right)^{2\Delta-2}$.

Proof. Since $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_t \subset V(G)$ are chosen randomly and independently, the events $A^1_{i,j}, \ldots, A^t_{i,j}$ are mutually independent. So

$$Pr(A_{i,j}) = \Big(Pr(A^l_{i,j})\Big)^t.$$

By the definition of $\overline{A^l_{i,j}}$ and Theorem 2, we have

$$Pr\left(\overline{A^l_{i,j}}\right) = Pr\left(E(F_l \cap H_i) \neq \varnothing, E(F_l \cap H_j) = \varnothing\right) \geq p^2(1-p)^{2\Delta}(1-\epsilon).$$

We set $\epsilon = \frac{1}{2\Delta}$. So $Pr\left(\overline{A^l_{i,j}}\right) \geq P_{k,\Delta}$, where

$$P_{k,\Delta} = \frac{1}{2k\Delta}\left(1-\frac{1}{2\Delta}\right)^{2\Delta-1}\left(1-\sqrt{\frac{1}{2k\Delta}\left(1-\frac{1}{2\Delta}\right)^{\Delta-1}}\right)^{2\Delta}.$$

Therefore, $Pr(A_{i,j}) = \left(Pr(A^l_{i,j})\right)^t \leq (1-P_{k,\Delta})^t$. ■

Now, we are ready to prove the main theorem.

Proof of Theorem 1. By Theorem 3, for every $1 \leq i \neq j \leq m$, $Pr(A_{i,j}) \leq (1-P_{k,\Delta})^t$.

Now we prove that if $t > \frac{\ln(4er_G(H))+\ln m}{\ln \frac{1}{1-P_{k,\Delta}}}$, then by Lovász Local Lemma, with positive probability no event $A_{i,j}$ occurs.

Construct the dependency graph whose vertices are the events $A_{i,j}$, where $1 \leq i,j \leq m$. Two events $A_{i,j}$ and $A_{i',j'}$ are adjacent if and only if $\Big(V(H_i) \cup V(H_j)\Big) \cap \Big(V(H_{i'}) \cup V(H_{j'})\Big) \neq \varnothing$. Recall that $r_G(H) = \max\limits_i r_G(H,H_i)$, where $r_G(H,H_i)$ is the number of subgraphs isomorphic to H whose intersection with H_i is nonempty. It is straightforward to verify that the maximum degree in the dependency graph is at most $4r_G(H)(m-1)$. Hence, if

$$t > \frac{\ln(4er_G(H)) + \ln m}{\ln \frac{1}{1-P_{k,\Delta}}},$$

then $e\,(1-P_{k,\Delta})^t\,\Big(4r_G(H)(m-1)+1\Big) < 1$, and by Lovász Local Lemma

$$Pr\Big(\bigcap_{i,j}\overline{A_{i,j}}\Big) > 0.$$

Therefore, if $t = 1 + \lceil \frac{\ln(4er_G(H))+\ln m}{\ln \frac{1}{1-P_{k,\Delta}}} \rceil$, then with positive probability no event $A_{i,j}$ occurs. So the random matrix C is a 1-disjunct matrix with positive probability. ■

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